Abstract—Demand for healthcare is increasing due to ever growing and aging population. Choosing an adequate schedule for medical staff can present a difficult dilemma for managers. The goal of nurse scheduling is to minimize the cost of the staff while maximizing their preferences and the overall benefits for the unit. This paper is focused on a new hybrid strategy based on detecting the optimal solution in nurse scheduling problem. The new proposed hybrid non-cyclic nurse scheduling combines randomly selected nurse and variable neighborhood descent search. The model is tested and obtained from a well-known previous published data-set.

Keywords—nurse scheduling problem; non-cyclic; variable neighborhood descent search; medical staff

I. INTRODUCTION

One of the most important factors that should be taken into consideration in organization management is human resource management within the organization for maximum efficiency at all times. Employee timetabling problems (ETPs) can naturally be represented by constraints networks for real world instances which are large and difficult.

For personnel management within hospitals, scheduling the nurses’ responsibilities is an important factor which is difficult to manage for maximum efficiency due to unknown number of patients each day, which makes it difficult to adequately and appropriately provide services to patients. The largest single cost factor in the hospital budget, typically representing about one third of the total, is the salaries of the nursing staff. If it were possible to make any inroads on the current escalation of hospital costs, it should begin with the most efficient possible utilization of the nursing staff [1]. On the other hand, it is also necessary to constantly aim toward minimizing labor costs, maximizing quality indicators and outcomes, and maintaining patient and employee satisfaction. In order to make the best staffing decisions and truly impact patient care it is necessary to have access to real time data. Inadequate staffing and schedule management are contributing factors to poor work environment, burnout and eventually turnover in any healthcare organization.

Nurse Scheduling Problem (NSP) represents a subclass of employee timetabling problems that are difficult to solve for optimal results. Studies of nurse scheduling problems date back to the early 1960s. Nurse scheduling deals with assigning shifts to staff nurses subject to satisfying required workload and other constraints. The constraints are classified into hard constraints (compulsory) and soft constraints (should be satisfied as much as possible). A feasible solution is a solution that satisfies all hard constraints. However, the quality of the duty roster is measured by satisfying the soft constraints and presents the most important thing in choosing an adequate schedule for nursing staff, but on the other hand it can be a difficult dilemma for nurse managers. Complete search algorithms, even with good heuristics are unable to solve large enough instances of ETPs. In fact, several local search techniques have been proposed in the past decade for solving timetabling problems. It has been shown that local search can efficiently solve large ETPs [2].

This paper is focused on new strategy based on hybrid approach to detecting the optimal solution in NSP. The new proposed hybrid approach is obtained by combining random nurse choice and variable neighborhood descent search. Also, this paper continuous the authors’ previous research in nurse decision-making, scheduling and rostering health-care organizations which are presented in [3] [4] [5] [6] [7].

The rest of the paper is organized in the following way: Section 2 provides an overview of the basic idea in NSP and related work. Section 3 presents the optimization problem and applied technique for solving nurse scheduling problem proposed in this paper. Experimental results are presented in Section 4, while Section 5 provides conclusions and some points for future work.
II. NURSE SCHEDULING PROBLEM AND RELATED WORK

The basic problem of NSP is to provide patient 24/7 using nurses who generally work five days a week, one shift per day, and prefer to have weekends off. Scheduling is usually done by nursing supervisors for the units of floors for which they are responsible. They estimate patient care requirements and allocate the available nursing staff to the days of the week so that these requirements are satisfied in general, and hospital personnel regulations observed. They try to schedule the nursing staff so that each nurse gets her share of weekends off and none of the nurses is rotated to evenings or night shifts for an unduly long time, and they also try to accommodate individual nurses’ requests for specific days off. And finally, preparation of the schedule is a time-consuming task for the nursing supervisor.

NSP is a well-known NP-hard scheduling problem that aims to allocate the required workload to the available staff nurses at healthcare organizations to meet the operational requirements and a range of preferences. The NSP is a two-dimensional employee timetabling problem that deals with the assignment of nursing staff to shifts across a scheduling period subject to certain constraints.

In general, there are two basic types of scheduling used for the NSP: cyclic and non-cyclic scheduling. In cyclic scheduling, each nurse works in a pattern which is repeated in consecutive scheduling periods, whereas, in non-cyclic scheduling, a new schedule is generated for each scheduling period: weekly, fortnightly or monthly. Cyclic scheduling was first used in the early 1970s due to its low computational requirements and the possibility for manual solution [8].

A. Related Work in Nurse Scheduling Problem

In the past decades, many approaches have been proposed to solve NSP as they are manifested in different models. In the 1990s, a number of papers provided classifications of nurse scheduling systems and reviews of methods for solving different classes of problems [9]. The three commonly used general methods are: mathematical programming (MP), heuristics and artificial intelligence (AI) approaches. Many heuristics approaches were straightforward automation of manual practices, which have been widely studied and documented [10] [11].

Further advances were made in applying linear and/or mixed integer programming and network optimization techniques for developing nurse rosters. Constraint programming (CP) methods were also used to model the complicated rules associated with nurse rosters. The methods were applied to problems involving cyclic and non-cyclic rosters. Typically, the problems contained roster rules applicable to a particular hospital. As such, these approaches may require substantial reformulation for use in a different hospital.

For combinatorial problems, exact optimization usually requires large computational times to produce optimal solutions. In contrast, metaheuristic approaches can produce satisfactory results in reasonably short times. In recent years, metaheuristics including: tabu search algorithm (TS), genetic algorithm (GA) and simulated annealing (SA) have all been proven as very efficient in obtaining near-optimal solutions for a variety of hard combinatorial problems including the NSP [12].

Some TS approaches have been proposed to solve the NSP. In TS, hard constraints remained fulfilled, while solutions move in the following way: calculate the best possible move which is not tabu, perform the move and add characteristics of the move to the tabu list. The TS with strategic oscillation used to tackle the NSP in a large hospital is presented in [13].

The basic idea is to find a genetic representation of the problem so that ‘characteristics’ can be inherited. Starting with a population of randomly created solutions, better solutions are more likely to be selected for recombination into novel GA solutions. In addition, these novel solutions may be formed by mutating or randomly changing the old ones [14].

III. OPTIMIZATION PROBLEM AND APPLIED TECHNIQUE

Optimization tools have greatly improved during the last two decades. This is due to several factors: (i) progress in mathematical programming theory and algorithmic design; (ii) rapid improvement in computer performances; (iii) better communication of innovative ideas and integration in widely used complex software. Consequently, many problems long viewed as out of reach are currently solved, sometimes in very moderate computing times. This success, however, has led researchers and practitioners to address much larger instances and more difficult classes of problems. Many of these may again only be solved heuristically.

A. Deterministic Optimization Problem

A deterministic optimization problem may be formulated as

$$\min \{ f(x) \mid x \in X, X \subseteq \mathcal{S} \}$$  \hspace{1cm} (1)

where $\mathcal{S}, X, x,$ and $f$ denote the solution space, the feasible set, a feasible solution, and a real-valued objective function, respectively. If $\mathcal{S}$ is a finite but large set, a combinatorial optimization problem is defined. If $\mathcal{S} = \mathbb{N}^n$, we refer to continuous optimization. A solution $x^* \in X$ is optimal if

$$f(x^*) \leq f(x), \quad x^* \in X$$  \hspace{1cm} (2)

An exact algorithm for problem (1), if one exists, finds an optimal solution $x^*$, together with the proof of its optimality, or shows that there is no feasible solution, or the solution is unbounded. Moreover, in practice, the time needed to do so should be finite and not too long. For continuous optimization, it is reasonable to allow for some degree of tolerance, to stop when sufficient convergence is detected.

Let is denote $N_k (k = 1, ..., k_{max})$, a finite set of pre-selected neighborhood structures, and with $N_k (x)$ the set of solutions in the $k$-th neighborhood of $x$. Most local search heuristics use only one neighborhood structure, $k_{max} = 1$. Often successive neighborhoods $N_k$ are nested and may be induced from one or
more metric (or quasi-metric) functions introduced into a solution space $S$. An optimal solution $x_{\text{opt}}$ (or global minimum) is a feasible solution where a minimum is reached. It is called a local minimum of (1) with respect to $N_k$, if there is no solution $x \in N_k(x') \subseteq X$ such that $f(x) < f(x')$. Metaheuristics, based on local search procedures, try to continue the search by other means after finding the first local minimum.

B. Variable Neighborhood Descent Search

Variable neighborhood search (VNS) is a metaheuristic proposed by some of the present authors a dozen years ago [15]. It is based on the idea of a systematic change of neighborhood both in a descent phase to find a local optimum and in a perturbation phase to get out of the corresponding valley [16]. Originally designed for approximate solution of combinatorial optimization problems, it was extended to address mixed integer programs, nonlinear programs, and recently mixed integer nonlinear programs.

VNS is based on three simple empirical facts: i) a local minimum with respect to one neighborhood structure is not necessarily so for another; ii) a global minimum is a local minimum with respect to all possible neighborhood structures; iii) local minima with respect to one or several neighborhoods, and thus a global optimum is more likely to be reached than with a single structure. Most local search heuristics use a single or sometimes two neighborhoods for improving the current solution ($k_{\text{max}} \leq 2$). Note that the final solution should be a local minimum, all $k_{\text{max}}$ neighborhoods, and thus a global optimum is more likely to be reached than with a single structure.

IV. MODELING THE NURSE SCHEDULING PROBLEM

Modeling the NSP is the process of ensuring that there are always enough nurses present, it comprises of numerous decisions based on different time horizons and different levels of details. These decisions can be divided into three planning phases, as illustrated in Fig. 1.

![Fig. 1. The three phases of nurse scheduling](image)

The long-term planning is a part of the overall strategic planning process for each ward. First the ward managers must estimate how many nurses with each of the necessary skills are needed during all possible time periods of the day. When the staffing demand is known and there is a given workforce of nurses, each nurse is assigned to a schedule specifying which shifts she should work, usually for a scheduling period of four to ten weeks. This phase in the planning process can be referred to as the mid-term planning, or nurse rostering. Whenever there is a shortage of nurses for a shift, the short-term planning consists of deciding whether to use overtime, to call in a nurse on her day off, to call in a substitute nurse, or to try to manage despite the shortage.

Depending on the context in which the schedule is to be used and how the scheduling process is carried out, a definition of a good schedule can differ. The two main categories of scheduling strategies are cyclic and non-cyclic, which will both be described below. A third strategy, which will also be commented on, is self-scheduling, a kind of non-cyclic scheduling that has quickly grown in popularity during the past two decades.

A. Cyclic schedules

Using a traditional cyclic schedule means repeating the same schedule over and over again until the ward decides to change the schedule. Each schedule is typically made for a period of 4 to 10 weeks and is used for 6-12 months. Since the same schedule is used repeatedly it is very important that it is almost perfectly fair with respect to the score, the distribution of unpopular shifts, the number of hours, and quality aspects. This choice of scheduling strategy imposes the boundary restriction that the first week of the schedule can follow on from the last week. Further, because the nurses are bound to use the same schedule repeatedly, they typically have a low level of influence on the scheduling when this kind of strategy is used.

B. Non-cyclic schedules

Non-cyclic scheduling means that a new schedule, usually for a period of 2-8 weeks, is created for each scheduling period.

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**Algorithm 1** Neighborhood change

Function NeighborhoodChange ($x, x', k$)

1. if $f(x') < f(x)$ then
2.   $x \leftarrow x'$ // Make a move
3.   $k \leftarrow k + 1$ // Initial neighborhood
else
4.   $k \leftarrow k + 1$ // Next neighborhood
end else

return $x, k$

---

**Algorithm 2** Variable neighborhood descent

Function VND ($x, k_{\text{max}}$)

1. $k \leftarrow 1$
2. repeat
3.   $x' \leftarrow \text{argmin}_{x \in N_{k_0}} f(x)$ // Find the best neighbor in $N_{k_0}$
4.   $x, k \leftarrow \text{NeighborhoodChange} (x, x', k)$ // Change neighborhood
5. until $k = k_{\text{max}}$

return $x$

The **variable neighborhood descent** (VND) method is obtained if a change of neighborhoods is performed in a deterministic way. It is presented in Algorithm 2, where neighborhoods are denoted as $N_k$, $k = 1, ..., k_{\text{max}}$. 
The advantage of a new schedule for each period is greater flexibility due to the possibility to take into account both changes on the ward and period specific requests from the nurses. A boundary condition is that the first week’s schedule is affected by the last week’s schedule from the previous scheduling period. Instead of making the schedule very fair in each period, it can be made reasonably fair and then information about for example the score, the distribution of unpopular shifts, and the number of hours worked can be passed on to next period, achieving a higher level of fairness in the long run.

C. Self-scheduling

Self-scheduling is a general term used for the kind of scheduling processes where the nursing staff is jointly responsible for creating the schedule. This kind of scheduling exists in different forms around the world, but in general involves the following steps: i) Without taking into account the staffing demand and other nurses’ preferences, each nurse individually proposes a schedule for herself; ii) An improved and more feasible schedule is created through informal negotiations between the nurses; iii) A scheduling group consisting of approximately four nurses makes further adjustments to the schedule; iv) The head nurse makes some final adjustments and approves the schedule.

D. Input data set

Table I gives the data set which is generated by the scheduling system presented in [18], where the normal shifts, eight hours a day, include the Day shift (D) (8 a.m. to 4 p.m.), the Evening shift (E) (4 p.m. to midnight), and the Night shift (N) (midnight to 8 a.m.).

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</table>

Special shifts, twelve hours a day, include the Daytime 8 o’clock shift (d) (8 a.m. to 8 p.m.) and the Nighttime 8 o’clock shift (n) (8 p.m. to 8 a.m.) The use of daytime and night-time 8 hour shifts is to make up for nursing shortages in exceptional cases. The Requested Day off by (R), and the Ordinary day off by (O). In addition, the second column denotes Shift Leader (L), and Member (M), respectively.

Based on the approximate ratio the total numbers of nursing personnel are 16, but 12 or 13 nurses should be on duty each day. According to percentages for three shifts, the department should have six, four, and three nurses on duty for the day, evening, and night shifts, respectively.

It is very easy to recognize that implemented data set [18] is very unbalanced when viewed in different shifts which is used in nurse scheduling. It can be shown for nurse A \( \{D, E, N\} = \{21, 1, 0\} \) and 8 Day off; contrary to nurse E \( \{D, E, N\} = \{6, 5, 11\} \) and 8 Day off, or for nurse O \( \{D, E, N\} = \{2, 7, 14\} \) and ? Day off.

V. Experimental Results

The focus of this research is to propose hybrid non-cyclic nurse scheduling model which combines randomly selected nurse and variable neighborhood descent search. Non-cyclic scheduling period is usually for a period of 2 weeks and, therefore the starting data-set is divided in two parts for 15 days each. For first half of a month (~2 weeks) one nurse schedule is generated and for second half of a month (~2 weeks) next nurse schedule which does not carry any consequence from the previous scheduling part is generated. On the other hand, the proposed hybrid scheduling model is at least very fair in each shift period and maintains high level of fairness during the timetabling in considering type on nurse shifts.

On May 8th “n” and “d” are shown in Table I., and every Requested Day off, and the Ordinary day off will be respected in creating new hybrid nurse scheduling timetable. It is typical for May 11th where 3 nurses are on Requested Day off and 2 nurses are on Ordinary day off.

Also, what is interesting to note is that typical work dynamic is: 5 nurses are in Day shift, 4 nurses are in Evening shift and 3 nurses are in Night shift. But, when necessary, this suggested schema is changed, therefore, for example, on May 14th: 5 nurses in D, 5 nurses in E, and 3 nurses in N; and on May 15th: 5 nurses in D, 3 nurses in E, and 3 nurses in N.

In order to balance the deficit between shifts, a score is calculated using the following formula:

\[
D_{\text{shift}} = |D_{\text{night}} - D_{\text{day}}| + |D_{\text{night}} - D_{\text{evening}}| + |D_{\text{day}} - D_{\text{evening}}|
\]

where \(D_{\text{shift}}\) represents the monthly gap between the quota and the number of nurses assigned by shift.

![Fig. 2. The variable neighborhood descent search](image)
TABLE II. EXPERIMENTAL RESULTS WITH NON-CYCLIC SCHEDULE APPROACH

<table>
<thead>
<tr>
<th>Nurse</th>
<th>Shifts</th>
<th>Shifts (%)</th>
<th>Dshift</th>
<th>Dshift [18]</th>
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<tbody>
<tr>
<td>A</td>
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<td>0.863</td>
<td>0.045</td>
<td>0.090</td>
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<td>B</td>
<td>B</td>
<td>0.727</td>
<td>0.181</td>
<td>0.090</td>
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<tr>
<td>C</td>
<td>C</td>
<td>0.545</td>
<td>0.181</td>
<td>0.272</td>
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<tr>
<td>D</td>
<td>D</td>
<td>0.727</td>
<td>0.454</td>
<td>0.000</td>
</tr>
<tr>
<td>E</td>
<td>E</td>
<td>0.363</td>
<td>0.409</td>
<td>0.227</td>
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<tr>
<td>F</td>
<td>F</td>
<td>0.652</td>
<td>0.260</td>
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<tr>
<td>G</td>
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<td>0.833</td>
<td>0.166</td>
<td>0.000</td>
</tr>
<tr>
<td>H</td>
<td>H</td>
<td>0.130</td>
<td>0.652</td>
<td>0.217</td>
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<td>I</td>
<td>I</td>
<td>0.217</td>
<td>0.695</td>
<td>0.087</td>
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<tr>
<td>J</td>
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<td>0.250</td>
<td>0.625</td>
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<td>K</td>
<td>K</td>
<td>0.291</td>
<td>0.416</td>
<td>0.291</td>
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<td>L</td>
<td>L</td>
<td>0.176</td>
<td>0.647</td>
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<td>P</td>
<td>0.250</td>
<td>0.100</td>
<td>0.650</td>
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</table>

The Table II. presents experimental results for nurse scheduling timetable when the proposed hybrid non-cyclic nurse scheduling which combines randomly selected nurse and variable neighborhood descent search is used. The experimental results present scheduling for period of 30 days with the attempt to balance between, Day, Evening and Night shifts with respect to every the Requested Day off, and the Ordinary day off as is given in the original input data [18].

The Table III. presents the deficit between shifts, which shows a score of imbalance between shifts calculated Dshift first for the hybrid non-cyclic nurse scheduling system proposed in this paper and second Dshift[18] which is calculated with original input data-set. In the table red bold presents the best experimental results, blue bold presents the second best experimental results, while pink bold presents the third best experimental results.

It is very easy to note that all Dshift are much less then Dshift[18], which guarantees better balance which is generated with the proposed novel hybrid Nurse Scheduling System. The average value of deficit between shifts for novel hybrid Nurse Scheduling System is 22.50, while Dshift[18] has much higher value of 38.75.

Also, it is very important to mention that schedule which is used as the input data-set is generated as "Scheduling nursing personnel on a microcomputer" [18] for hospital, and that developed software and implemented system is promoted and used at a leading hospital in Taiwan.

VI. CONCLUSION AND FUTURE WORK

One challenge when working with a new ward is to understand what is essential in their scheduling, and to be able to successfully deliver a schedule, it is of crucial importance to understand their values and traditions. During this type of work, the responses from the nurses have usually been expectant and skeptical. Expectant because of the time-consuming work and difficulties associated with the manual scheduling process, and skeptical mainly because they are afraid of losing control over the scheduling. Because of the nurses’ skepticism, it is important to present the outcome of the automatic scheduling pedagogically and to emphasize that the optimization tool only offers a qualified suggestion for a schedule. If it is considered beneficial for the nurses to be allowed to make minor adjustments themselves.

The aim of this paper is to propose the new hybrid strategy, the novel hybrid non-cyclic nurse scheduling system for detecting the quality solution in nurse scheduling problem. The new proposed hybrid approach is obtained by combining randomly selected nurse and variable neighborhood descent search. The model is tested with original real-world data-set from leading hospital in Taiwan.

The great benefit of this approach should be the time and effort saved if the head nurse is handed a schedule that is both feasible and fair. Other benefits are the objectivity of a computerized planning tool and the decrease in lead time for constructing a schedule.
Preliminary experimental results of our research, compared with original input data-set from leading hospital in Taiwan, are better, which is shown in this paper. The experimental results encourage further research. Our future research will focus on creating new hybrid model combined some new soft-computing techniques, to solve problems logically, considering different options until the best solution is discovered, which will efficiently solve NSP. The new model will be tested with original real-world dataset for longer periods, including the data-set for 2018 obtained from the Oncology Institute of Vojvodina in Serbia.

REFERENCES